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Covariant Actions for the Bosonic Sector of D=10 IIB Supergravity

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Abstract

Covariant actions for the bosonic fields of D=10 IIB supergravity are constructed with the help of a single auxiliary scalar field and in a formulation with an infinite series of auxiliary (anti)–self–dual 5-form fields.

The construction of a complete action for chiral $N = 2$ (or IIB) supergravity in ten–dimensional space–time [1] is a long standing problem, which has become topical again in connection with a "duality revolution". Its solution has been hampered by the presence in the bosonic sector of the theory of a four–form gauge field $A^{(4)}(x)$ whose five–form field strength $M = dA^{(4)} + ...$ is self–dual, i.e. $M = M^*$ (where * denotes the Hodge conjugation of the differential forms). This field comes from the Ramond–Ramond (RR) sector of IIB superstring theory. The problem is to find a $D = 10$ covariant action from which the self–duality condition would follow as an $A^{(4)}$ field equation of motion.

An action proposed for IIB supergravity in [2] is based on a modification of the Siegel approach [3] to a space–time covariant Lagrangian description of self–dual fields. An

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unsatisfactory feature of this construction is that Lagrange multiplier fields which are used to get the self–duality condition cannot be completely eliminated by local symmetries and equations of motion of the model, and the treatment of remaining auxiliary degrees of freedom is not clear.

Another possibility of solving the problem of the self–dual field is to use a covariant McClain–Wu–Yu approach which involves infinite number of auxiliary fields [4, 5, 6]. An interesting observation concerning this construction is that it arises as an effective action of a closed superstring field theory [5]. Because of an infinite series of fields one may encounter technical complications when generalizing such a model to a locally supersymmetric case[1].

In this note we propose a $D = 10$ covariant action for the bosonic sector of IIB supergravity, where to get manifest space–time covariance of the self–dual field part of the action only one scalar auxiliary field is used. Such a covariant approach to deal with self–dual fields was proposed in [8, 9] and has been applied to the duality–symmetric description of (super)Maxwell theory [8] coupled to electric and magnetic sources [10], and to the $SL(2, R) \times SO(6, 22)$ invariant effective field theory action in $D = 4$ of a toroidally compactified heterotic string [8]. This approach has proved to be pretty helpful in constructing a complete action for the M–theory 5–brane propagating in a $D = 11$ supergravity background [11] and considering its K3 compactification [12]. A relationship between the single scalar field approach and the McClain–Wu–Yu approach has been established in [9], which allows one to straightforwardly get a IIB supergravity action in the latter formulation. We present this action in the conclusion of the article.

The covariant action for the bosonic sector of $D = 10$ IIB supergravity which one gets with the use of a single scalar auxiliary field has the following form

$$\begin{aligned} S = \int d^{10}x\sqrt{-g}[R - 2\partial_m\phi\partial^m\phi - 2e^{2\phi}\partial_m\phi'\partial^m\phi' - \frac{1}{3}e^{-\phi}H_{lmn}H^{lmn} \\ - \frac{1}{3}(H'_{lmn} - \phi' H_{lmn})(H'^{\,lmn} - \phi' H^{\,lmn})] \\ -\frac{1}{6}\int d^{10}x\frac{\sqrt{-g}}{\partial_r a\partial^r a}\partial^l a(x)M^*_{lm_1\ldots m_4}\mathcal{M}^{m_1\ldots m_4 p}\partial_p a(x) - 4\int A^{(4)}\wedge H\wedge H', \end{aligned} \tag{1}$$

where $g_{mn}(x)$ (m,n=0,...,9) is a $D = 10$ space–time metric, $R(x)$ is a $D = 10$ scalar curvature, $\phi(x)$ and $\phi'(x)$ are, respectively, the Neveu–Schwarz (NS) dilaton and the RR scalar; $H = dB$ and $H' = dB'$ are the three–form field strengths of, respectively, the NS two–form gauge field $B(x)$ and the RR two–from gauge field $B'(x)$,

$$M = dA^{(4)} + \frac{1}{2}B\wedge H' - \frac{1}{2}B'\wedge H \tag{2}$$

[1]Note, however, that in [7] it has been shown how to construct in this approach superfield actions for $D = 4$ supersymmetric Maxwell theory with manifest dualities.

is the five–form field strength of the RR gauge field $A^{(4)}(x)$ extended with the Chern–Simons–like term constructed of the two–form fields, and $\mathcal{M} = M - M^*$ is the anti–self–dual part of $M(x)$. Finally $a(x)$ is the auxiliary scalar field ensuring the manifest $D = 10$ covariance of the $A^{(4)}$–part of the action, which would otherwise be lost (see [13] and references therein). Entering the action in a nonpolynomial way the field $a(x)$ points to a nontrivial topological structure of the self–dual field configurations. The form of the $M(x)$ terms as written in (1) is convenient for making the symmetry analysis of the action and getting the equations of motion, though, one can also present the $M(x)$ part of the action (1) in more conventional form

$$S_M = -\int d^{10}x\sqrt{-g}[\frac{1}{60}M_{m_1\ldots m_5}M^{m_1\ldots m_5} - \frac{1}{12\partial_r a\partial^r a}\partial^l a(x)\mathcal{M}_{lm_1\ldots m_4}\mathcal{M}^{m_1\ldots m_4 p}\partial_p a(x)]$$
$$-4\int A^{(4)}\wedge H\wedge H' \tag{3}$$

which is the sum of the kinetic term of $A^{(4)}$, the term quadratic in the anti–self–dual tensor $\mathcal{M}(x)$ and the Chern–Simons term. In this second form (3) the action (1) is the same as the one written in [14] except that it contains the second term in (3). This difference is crucial since, while in [14] the self–duality condition

$$\mathcal{M} = M \ - \ M^* = 0 \tag{4}$$

had to be imposed into the theory "by hand", with this modification Eq. (4) becomes the consequence of the $A^{(4)}$ equations of motion. This has been demonstrated in detail for various models containing self–dual fields [8]–[13] and we address the reader to these references. Let us only note that, in addition to $D = 10$ general coordinate transformations, standard gauge transformations of B, B' and $A^{(4)}$ and global $SL(2,\mathrm{R})$ duality mixing of ϕ and ϕ' and of B and B' [1, 14], the action (1) is also invariant under the following local transformations of $A^{(4)}$ and $a(x)$:

$$\delta A^{(4)} = da\wedge\varphi^{(3)}, \qquad \delta a(x) = 0; \tag{5}$$

$$\delta a(x) = \varphi(x), \qquad \delta A^{(4)}_{mnpq} = \frac{\varphi}{(\partial a)^2}\mathcal{M}_{mnpqr}\partial^r a. \tag{6}$$

where $\varphi^{(3)}(x)$ is a three–form and $\varphi(x)$ is a scalar gauge parameter.

The local symmetry of the action under the transformations (5) reflects the fact that $A^{(4)}$ is self–dual on the mass shell, i.e. it contains twice less (namely 35) physical degrees of freedom in comparison with an ordinary four–form abelian field in $D = 10$.

The transformations (6) allow one to gauge fix $a(x)$ at the expense of the manifest $D = 10$ covariance [8, 9] of the action (1). Note that the gauge fixing of (6) where $\partial_m a$ would be lightlike (i.e. $(\partial a)^2 = 0$) is inadmissible because of the presence of the norm of the vector $\partial_m a$ in the denominator of the action (1) or (3). Thus globally $da(x)$ is a closed form but not exact.

An interesting feature of the symmetries (5) and (6) is that, as in the case of the M–theory five–brane [11], they fix the relative coefficients of the term containing the auxiliary field $a(x)$ and of the Chern–Simons (the last) term in the action (1).

The equations of motion one gets from the action (1) are the same as the covariant equations of motion of the bosonic fields of $D=10$ IIB supergravity which have been known for a long time [1]. The field $a(x)$ disappears from them. As one can directly see from (3) the $a(x)$ dependent terms in the field equations are always proportional to the anti–self–dual tensor $\mathcal{M}(x)$ which vanishes on the mass shell (Eq. (4)).

For completeness we also present how the self–dual part of the IIB supergravity action looks like in a simplified covariant formulation with an infinite number of auxiliary 5-form fields $\Lambda^{(n)}_{m_1\ldots m_5}$ $(n=1,\ldots\infty)$ self–dual for odd n and anti–self–dual for even n:

$$S_M = -\int d^{10}x\sqrt{-g}\left[\frac{1}{60}M_{m_1\ldots m_5}M^{m_1\ldots m_5} - \Lambda^{(1)}_{m_1\ldots m_5}M^{m_1\ldots m_5} + \sum_{n=0}^{\infty}(-1)^n\Lambda^{(n+1)}\Lambda^{(n+2)}\right] \quad (7)$$

$$-4\int A^{(4)}\wedge H\wedge H'.$$

As it has been shown in detail for free self–dual fields in $D=2p+2$ [4]–[7], there is an infinite parameter local symmetry (valid also when $A^{(4)}$ couples to B and B' as in (7)) which allows one (at the level of equations of motion) to eliminate all the Lagrange multipliers and remain with the self–duality condition (4).

In [9] it has been observed that a self–dual action of the form (3) (or (1)) can be viewed as a consistent covariant truncation (or a gauge fixing of the infinite–parameter local symmetry) of (7), where all $\Lambda^{(n)}$ with $n>1$ are set equal to zero and

$$\Lambda^{(1)}_{m_1\ldots m_5} = \frac{1}{6\partial_r a\partial^r a}\left(\partial^l a(x)\mathcal{M}_{l[m_1\ldots m_4}\partial_{m_5]}a(x) + \frac{1}{4!}\epsilon_{m_1\ldots m_5 m_6\ldots m_{10}}\partial_l a(x)\mathcal{M}^{l m_6\ldots m_9}\partial^{m_{10}}a(x)\right).$$

It should be possible to promote the action (1) to a complete IIB supergravity action, where off–shell supersymmetric transformations of fermionic fields will include nonconventional terms proportional to the anti–self–dual tensor $\mathcal{M}$, which vanish when Eq. (4) is taken into account, as the example of the duality symmetric super–Maxwell model teaches us [13, 8].

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